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Low cost curvature correction of bandgap references for integrated sensors

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Abstract

Due to increasing demands on the accuracy of integrated sensors, it is very important to improve the accuracy of bandgap voltage references. Since the second order non linearity of the function $V_{\mathrm{BE}}(T)$ is generally the main limit to the accuracy of calibrated bandgap voltage references, several methods for the curvature-correction of $V_{\mathrm{BE}}(T)$ have been reported in literature; unfortunately these methods require quite complex circuitry. In this paper we investigate a low cost curvature correction method, consisting in taking advantage of the temperature dependence of integrated resistors and in using PTAT/R collector currents; furthermore we introduce two new circuit topologies which permit to take full advantage of this approach in spite of technological limitations.

Keywords: Integrated sensors; Bandgap voltage references; Curvature correction

1. Introduction

1.1. Non linearity and curvature of $V_{BE}(T)$

Measuring is to compare an unknown quantity and a reference quantity; for this reason any measurement system must contain at least one reference; in integrated measurement systems, among all possible electrical references, voltage references are generally the most useful and, among all possible voltage references, bandgap voltage references are the most accurate.

Bandgap voltage references make use of the temperature dependence of the base to emitter voltage of bipolar transistors which is best described by the Meijer model [1] (here we only notice that, although the physical interpretation of the parameters is different, the analytical expressions given by the Meijer model are the same as those given by the Gummel–Poon model)

$$V_{\mathrm{BE}}(I_{\mathrm{C}},\ T) = \frac{kT}{q}\ln\left[\frac{I_{\mathrm{C}}}{I_{\mathrm{S}}(T)}\right],\qquad I_{\mathrm{S}}(T) = C_M T^{\eta_M} \mathrm{e}^{\frac{-qV_M}{kT}} \tag{1}$$

(unless differently stated we refer to npn transistors and, for convenience, the collector currents of npn (pnp) transistors are considered positive (negative) when they enter the collector terminal).

From (1) we have

$$V_{\mathrm{BE}}(T) = V_M + \frac{kT}{q}\ln\left(\frac{I_{\mathrm{C}}}{C_M}\right) - \eta_M\frac{kT}{q}\ln(T) \tag{2}$$

From (2), assuming that the collector current is independent on temperature (which is not the case in most applications), we can compute the derivatives of the base–emitter voltage respect to temperature

$$\frac{\partial^n V_{\mathrm{BE}}(T)}{\partial T^n} = \begin{cases} \dfrac{k}{q}\left\{\ln\left(\dfrac{I_{\mathrm{C}}}{C_M}\right) - \eta_M[\ln(T)+1]\right\} & n = 1 \\ \dfrac{\eta_M k}{q}\dfrac{(-1)^{n-1}(n-2)!}{T^{n-1}} & n > 1 \end{cases} \tag{3}$$

and we can therefore write the Taylor polynomial of Nth order of $V_{\mathrm{BE}}(T)$ in case of constant collector current

$$V_{\mathrm{BE}}(T) \cong V_{\mathrm{BE}}(T_0) + \left\{\frac{k}{q}\ln\left(\frac{I_{\mathrm{C}}}{C_M}\right) - \eta_M\frac{k}{q}[\ln(T_0)+1]\right\} \times (T-T_0) + \sum_{n=2}^{N}\frac{(-1)^{n+1}}{n(n-1)}\left(\frac{\eta_M k}{q}\frac{1}{T_0^{n-1}}\right)(T-T_0)^n \tag{4}$$

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Although (4) provides a simple expression for the Taylor polynomial of any order of $V_{BE}(T)$, the Lagrange's formula of the error of the Taylor polynomial shows that the error, when non linear terms of order higher than the second are neglected, is very small and negligible for many applications [1] (for instance less than $28.72 \times \eta_M$ μV in the temperature range [250, 350 K]); even if this result has been derived only for a constant collector current, it somehow holds for the "practical" collector currents used in integrated circuits, so that the non linear terms of order higher than the second are important only for very high accuracy applications and/or for very large temperature ranges. For this reason (the second order derivative gives the, by far, dominant non linear error) the reduction of the non linearity of $V_{BE}(T)$ is generally referred to as "curvature correction"; we will also adopt this nomenclature (on the contrary the expression "third order curvature correction" [2,3] is not correct since, strictly, the curvature is related to the second order derivative).

1.2. Existing curvature correction and non-linearity correction methods

The bandgap voltage references are typically obtained by adding, with properly chosen coefficients, a base to emitter voltage, $V_{BE}(T)$, and a PTAT (proportional to absolute temperature) voltage. PTAT voltages may be easily generated in integrated circuits by biasing two matched transistors with two currents whose ratio, r, is temperature independent and by taking the difference between two base to emitter voltages, so that the voltage

$$\Delta V_{BE}(T) = V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) - V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right) = V_T \ln\left(\frac{I_{C2}}{I_{C1}}\right) = V_T \ln(r) \quad (5)$$

is PTAT.

In practical circuits the $\Delta V_{BE}(T)$ voltages are quite accurately proportional to the absolute temperature (and then they are linearly temperature dependent); on the contrary, the non linearity of $V_{BE}(T)$ constitutes the main limit to thermal stability of calibrated bandgap references. For this reason many techniques for the correction of the non linearity of $V_{BE}(T)$ have been presented.

In [4] it has been reported (without explanation) that a reduction of the non linearity of $V_{BE}(T)$ is obtained if PTAT collector currents are used; we also stress that in most bandgap circuits a PTAT/R collector current is used (the expression PTAT/R [5] puts in evidence that the collector current is not really PTAT since R depends on temperature).

It is also possible to reduce the non linearity of $V_{BE}(T)$ by taking advantage of the temperature dependence of integrated resistors, which is generally expressed by mean of the relative temperature coefficient (TC) of resistors, defined as follows

$$\mathrm{TC} = \frac{1}{R}\frac{\partial R}{\partial T} \quad (6)$$

In [6] a PTAT current (generated by applying a PTAT voltage to a thermally stable resistor, TC = 0) is injected into a temperature dependent resistor therefore producing a non linearly temperature dependent voltage, which is used to compensate the non linearity of $V_{BE}(T)$; an improved version of this technique was presented in [2] where the shunt connection of two different resistor types, each with its own TC and linearly temperature dependent, was used to produce a non linearly temperature dependent resistor, so that higher order non linear terms may also be compensated.

A similar approach consists in the generation of a reference current (a reference voltage is applied to a thermally stable resistor); the reference current is injected through resistors with intrinsic non linear temperature dependence (for instance lightly doped drain diffused resistors) in order to generate the non linearly temperature dependent voltage required to compensate the non linearity of $V_{BE}(T)$ [7,8].

Recently, it has been noted that simply using in a standard Brokaw cell (or in similar circuit topologies) resistors with negative TC "instead of the usual positive TC resistors, . . . , vastly improves the curvature of the bandgap circuit" [9]; this improvement was accomplished by just properly selecting the resistor type to be used in standard circuit topologies.

Many other techniques have been presented, such as: to employ the temperature dependence of the current gain of bipolar transistors [10]; to use the $I_D(V_{DS})$ relation of MOSFET [11–13]; to use a piecewise linear correction voltage [14,15]; to generate currents proportional to higher powers of the absolute temperature by using PTAT currents (generated by applying a PTAT voltage to a thermally stable resistor) and translinear circuits [16]; to use a collector current $I_C = C_X T^{\eta_M}$ so that the base to emitter voltage becomes a linear function of the temperature (since the direct implementation of this idea is rather difficult, more practical implementations have been proposed [17–19]).

Although several techniques are available, all these techniques generally require quite complex circuitry and calibration, which may be unacceptable in applications where low cost and small area are fundamental issues.

1.3. Organization of the paper

Among the methods above mentioned, using PTAT/R collector currents and taking advantage of the temperature dependence of integrated resistors are very convenient for low cost integrated systems because they do not require any additional circuitry and may be implemented using standard circuit topologies. Although both these approaches are yet known, a satisfactory theoretical analysis of their potentialities and limits has not yet been reported, resulting in sub-optimal curvature correction.

In this paper we present a study on the effects on curvature of the temperature dependence of collector currents.

Since optimal curvature correction would typically require resistors with very large absolute values of the TC (not available in many processes), we introduce two new circuit topologies which overcome this technological limitation by mean of a virtual resistor (with high absolute value of the TC); the use of virtual resistors requires calibration, and therefore a single temperature autocalibration procedure is described and the consequences of the spread of TCs of integrated resistors are discussed.

2. The effect of the temperature dependence of the collector current on the curvature of $V_{BE}(T)$

2.1. Non linear error and total non linear error

In order to quantify the non linearity of $V_{BE}(T)$ in a given temperature range (TR), we define the non linear error as the difference between $V_{BE}(T)$ and its (least squares) linear fit computed in the given TR.

Furthermore we define the total non linear error in TR as the difference between the maximum and the minimum non linear errors in TR.

2.2. Determination of the first, second and third order derivatives of $V_{BE}(T)$ in case of temperature dependent collector current

In most practical cases the collector current is somehow temperature dependent; in this general case we obtain

$$\frac{\partial V_{BE}(T)}{\partial T}=\frac{k}{q}\left\{\ln\left(\frac{I_C}{C}\right)+T\frac{1}{I_C}\frac{\partial I_C}{\partial T}-\eta_M\left[\ln(T)+1\right]\right\}$$
$$\frac{\partial^2 V_{BE}(T)}{\partial T^2}=\frac{k}{q}\left\{2\frac{1}{I_C}\frac{\partial I_C}{\partial T}+T\left[\frac{-1}{I_C^2}\left(\frac{\partial I_C}{\partial T}\right)^2+\frac{1}{I_C}\frac{\partial^2 I_C}{\partial T^2}\right]-\eta_M\frac{1}{T}\right\}$$
$$\frac{\partial^3 V_{BE}(T)}{\partial T^3}=\frac{k}{q}\left\{\frac{-3}{I_C^2}\left(\frac{\partial I_C}{\partial T}\right)^2-\frac{3}{I_C}\frac{\partial^2 I_C}{\partial T^2}+T\left[\frac{2}{I_C^3}\left(\frac{\partial I_C}{\partial T}\right)^3-\frac{3}{I_C^2}\frac{\partial I_C}{\partial T}\frac{\partial^2 I_C}{\partial T^2}+\frac{1}{I_C}\frac{\partial^3 I_C}{\partial T^3}\right]+\eta_M\frac{1}{T^2}\right\} \quad (7)$$

From (7) it may be observed that, even in case of temperature dependent collector currents, the non linear terms do not depend on the collector current level, that is they do not change if the collector current is multiplied by a constant (in fact, if we subtract two V_{BE} voltages generated by applying to two matched transistors two proportional currents, the non linear terms are cancelled and a PTAT voltage is obtained).

2.3. Analysis of the curvature of $V_{BE}(T)$ in case of collector current generated by applying a linearly temperature dependent to a linearly temperature dependent voltage

Let us consider a collector current obtained by applying a linearly temperature dependent voltage V to a linearly temperature dependent resistor R; in general we may write

$$I_C=\frac{V_0[1+\alpha(T-T_0)]}{R_0[1+\beta(T-T_0)]} \quad (8)$$

where α and β are the TCs at the temperature T_0 of, respectively, V and R. It may then be found

$$\frac{\partial^2 V_{BE}(T_0)}{\partial T^2}=\frac{k}{q}\left\{2(\alpha-\beta)+T_0(\beta^2-\alpha^2)-\frac{\eta_M}{T_0}\right\} \quad (9)$$

$\partial^2 V_{BE}(T_0)/\partial T^2$ (and therefore the curvature of $V_{BE}(T)$) may be zeroed if and only if

$$\alpha^2T_0^2-2\alpha T_0+\eta_M+2\beta T_0-\beta^2T_0^2=0 \quad (10)$$

For simplicity it is better to suppose that only one resistor type, with its own TC, β, is available (this hypothesis will be removed later on), so that $\partial^2 V_{BE}(T_0)/\partial T^2$ is a function of α and, in particular, is a parabola $p_1(\alpha)$; as a consequence the following two cases are possible:

1. The parabola $p_1(\alpha)$ may be zeroed for the values

$$\alpha_{1,2}=\frac{2T_0\pm\sqrt{4T_0^2-4T_0^2(\eta_M+2\beta T_0-\beta^2T_0^2)}}{2T_0^2}=\frac{1\pm\sqrt{1-(\eta_M+2\beta T_0-\beta^2T_0^2)}}{T_0} \quad (11)$$

However, since α must be a real number, $\partial^2 V_{BE}(T_0)/\partial T^2$ may only be zeroed if

$$1-(\eta_M+2\beta T_0-\beta^2T_0^2)=\underbrace{\beta^2T_0^2-2\beta T_0+1-\eta_M}_{c}\geq 0$$
$$\beta\leq\frac{1-\sqrt{\eta_M}}{T_0}\quad\text{or}\quad\beta\geq\frac{1+\sqrt{\eta_M}}{T_0} \quad (12)$$

For typical values of the Meijer parameter η_M (3–6), both these conditions are quite hard to be accomplished; for instance

$$\begin{aligned}&[\eta=3,\ T_0=300\,\mathrm{K}]\rightarrow[\beta\leq-2433\,\mathrm{ppm/K}\quad\text{or}\\&\quad\beta\geq 9106\,\mathrm{ppm/K}]\\&[\eta=6,\ T_0=300\,\mathrm{K}]\rightarrow[\beta\leq-4831\,\mathrm{ppm/K}\quad\text{or}\\&\quad\beta\geq 11,498\,\mathrm{ppm/K}]\end{aligned} \quad (13)$$

2. The parabola $p_1(\alpha)=\partial^2 V_{BE}(T_0)/\partial T^2$ may not be zeroed; in this case it is evident from (9) that $p_1(\alpha)\leq 0\forall\alpha\in R$ and the minimum of $|p_1(\alpha)|=|\partial^2 V_{BE}(T_0)/\partial T^2|$ occurs for $\alpha=1/T_0$, so that the voltage

$$V(T) = V_0(1 + \alpha(T - T_0)) = V_0\left[1 + \frac{1}{T_0}(T - T_0)\right] = V_0\frac{T}{T_0} \tag{14}$$

is a PTAT voltage (and therefore the collector current is a PTAT/R current)

We stress that, if the conditions (12) may not be satisfied, even in presence of a non linear temperature dependence of the resistors, it is anyway convenient, from the point of view of the curvature correction (i.e. minimisation of the second order derivative), to use PTAT/R collector currents; in fact, if we consider

$$I_C = \frac{V_0[1 + \alpha(T - T_0)]}{R_0[1 + \beta_1(T - T_0) + \beta_2(T - T_0^2)]} \tag{15}$$

then we find

$$\frac{\partial^2 V_{BE}(T_0)}{\partial T^2} = \frac{k}{q}\left\{2(\alpha - \beta_1) + T_0(\beta_1^2 - \alpha^2 - 2\beta_2) - \frac{\eta}{T_0}\right\} \tag{16}$$

and, again, elementary geometric considerations on the parabola $p_2(\alpha) = \partial^2 V_{BE}(T_0)/\partial T^2$ show that, if it is not possible to zero $p_2(\alpha)$, the minimum of $|p_2(\alpha)|$ occurs for $\alpha = 1/T_0$ (that is when the collector current is a PTAT/R current).

We notice that in fact PTAT/R currents are used in most bandgap reference circuits; however the reduction of the curvature deriving from this choice has been mentioned only in [4,20], but without explanation and with reference only to true PTAT currents (we proved that it also holds for PTAT/R currents, that is currents generated by applying a PTAT voltage to a temperature dependent resistor).

2.4. Selection of the best resistor type for curvature correction

In case the conditions (12) may not be satisfied it is also important to identify which kind of resistor type, among the many normally available, should be used in order to minimise the curvature. Recently the following experimental result has been reported (without explanation): using, in a standard Brokaw cell (or in similar circuit topologies), resistors with negative TC "instead of the usual positive TC resistors, . . . , vastly improves the curvature of the bandgap circuit" [9]. We want to give reasons of this experimental result and to show how to select the best resistor type (from the point of view of curvature correction).

In most cases different kinds of resistor types, each with a different TC, are available but no resistor type satisfies (12); however, since for any of these resistor types it is convenient to use a PTAT/R collector current, we may substitute $\alpha = 1/T_0$ in (9) obtaining

$$\frac{\partial^2 V_{BE}(T_0)}{\partial T^2} = \frac{k}{q}\left\{T_0\beta^2 - 2\beta + \left(\frac{2 - 1 - \eta_M}{T_0}\right)\right\} \tag{17}$$

As a result $\partial^2 V_{BE}(T_0)/\partial T^2$ is a parabola $p_3(\beta)$ which assumes the zero value for

$$\beta = \frac{1 \pm \sqrt{1 - T_0(1 - \eta_M/T_0)}}{T_0} = \frac{1 \pm \sqrt{\eta_M}}{T_0} \tag{18}$$

As we discussed, for typical values of the Meijer parameter η_M, the conditions (12) (and therefore also the conditions (18)) may not be satisfied in most processes; however elementary geometric considerations show that in this case, if many resistor types, each with its own TC, β_k, are available, then the minimum of $|p_3(\beta)| = |\partial^2 V_{BE}(T_0)/\partial T^2|$ occurs when β_k minimises the quantity d_k defined as follows

$$d_k = \min\left\{\left|\beta_k - \frac{1 - \sqrt{\eta_M}}{T_0}\right|, \quad \left|\beta_k - \frac{1 + \sqrt{\eta_M}}{T_0}\right|\right\} \tag{19}$$

If, for instance, we assume the typical values

$$\eta = 3, \quad T_0 = 300\,\mathrm{K} \tag{20}$$

then the second order non linearity (curvature) of $V_{BE}(T)$ may be zeroed by using resistors with

$$\mathrm{TC}_{\mathrm{ideal\_1,2}} = \beta_{1,2} = \frac{1 \pm 2}{T_0} = \begin{cases} -3333.3\,\mathrm{ppm/K} \\ 10,000\,\mathrm{ppm/K} \end{cases} \tag{21}$$

On the other hand, if we only have resistors with

$$\begin{aligned} \beta_{R_1} &= \mathrm{TC}_{R_1} = 1000\,\mathrm{ppm/K} \\ \beta_{R_2} &= \mathrm{TC}_{R_2} = 600\,\mathrm{ppm/K} \\ \beta_{R_3} &= \mathrm{TC}_{R_3} = -600\,\mathrm{ppm/K} \\ \beta_{R_4} &= \mathrm{TC}_{R_4} = -1000\,\mathrm{ppm/K} \end{aligned} \tag{22}$$

the best possible curvature correction (within this method) is achieved if the collector current is generated by applying a PTAT voltage to a resistor of the fourth group. In most processes, for typical values of the Meijer parameter (η_M, the best resistor type will be the one with the most negative TC (in agreement with the experimental results reported in [9]).

Figs. 1–3 show the typical non linear errors (the parameter η_M has been set to 3) obtained by using different collector currents in the temperature range [250, 350 K]. In all these figures the solid line refers to a constant collector current and the dashed lines refer to PTAT/R collector currents where the TC of the resistor changes (−5000, −2500 and 0 ppm/K in Fig. 1; 0, 2500 and 5000 ppm/K in Fig. 2; 5000, 7500 and 10,000 pm/K in Fig. 3).

From Fig. 1 it is evident that if a PTAT/R collector current is used with a TC equal to −2500 ppm/K the curvature is strongly reduced.

Fig. 4 shows the total non linear error (above defined) as a function of the TC of resistances. It is evident the presence of two minima, but it is clear that it is impossible to zero the total non linear error (in fact, even if $\partial^2 V_{BE}(T_0)/\partial T^2 = 0$, other higher order non linear terms are present).

Let us now suppose that we are able to generate a PTAT/R collector current using a resistor whose TC may be

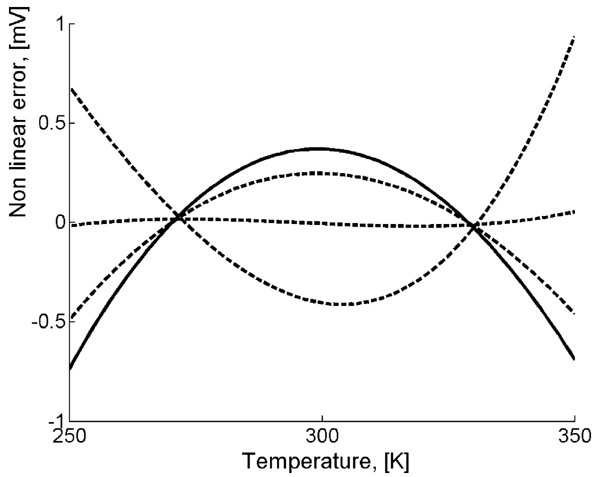

Fig. 1. The non linear error as a function of the temperature for a constant collector current (solid line) and for different PTAT/R collector currents (dashed lines; the TC of the resistor varies from −5000 up to 0 ppm/K); the Meijer parameter η_M has been considered equal to 3.

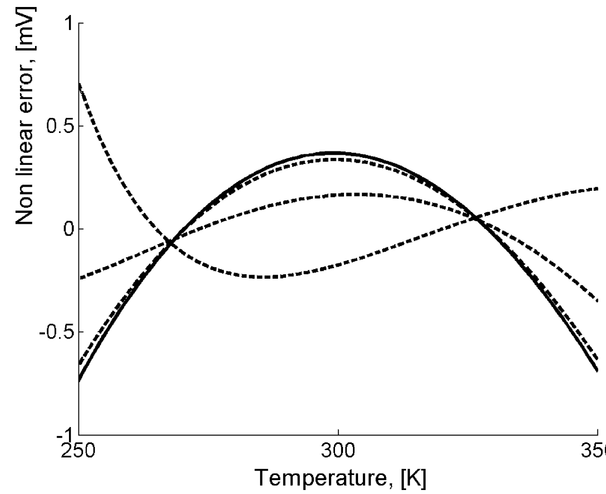

Fig. 3. The non linear error as a function of the temperature for a constant collector current (solid line) and for different PTAT/R collector currents (dashed lines; the TC of the resistor varies from 5000 up to 10,000 ppm/K); the Meijer parameter η_M has been considered equal to 3.

arbitrarily defined (later on we will discuss possible circuit solutions); from Fig. 4 it is evident that the two solutions (14) are not equivalent (in one case the total non linear error is much smaller). Another significant difference between the two solutions is the tolerance on TCs which permits to achieve a predefined performance; for instance, if the total non linearity error must be kept below 0.5 mV we should satisfy one of the two conditions

$$\begin{aligned} A &\equiv (-3500\,\text{ppm/K} \le \beta \le -1000\,\text{ppm/K}) \\ B &\equiv (7500\,\text{ppm/K} \le \beta \le 9300\,\text{ppm/K}) \end{aligned} \tag{23}$$

so that, beside better performance (as it is clear from Fig. 4), the solution A also gives better results from the point of view of rejection of spread of TCs. We note that (23) are in agreement (discrepancies are due to higher order non linearities) with the theoretical values

$$\beta = \frac{1 \pm \sqrt{\eta}}{T_0} = \frac{1 \pm \sqrt{3}}{T_0} = \begin{cases} -2440\,\text{ppm/K} \\ 9106\,\text{ppm/K} \end{cases} \tag{24}$$

The derivatives of order higher than the second may also introduce non negligible errors; we may compute

$$\frac{\partial^3 V_{\text{BE}}(T_0)}{\partial T^3} = \frac{k}{q}\left\{2T_0(\alpha^3 - \beta^3) - 3(\alpha^2 - \beta^2) + \frac{\eta}{T_0^2}\right\} \tag{25}$$

If the collector current is a PTAT/R current, then we have $\alpha = 1/T_0$, so that

$$\frac{\partial^3 V_{\text{BE}}(T_0)}{\partial T^3} = \frac{k}{q}\left\{-2T_0\beta^3 + 3\beta^2 + \frac{\eta - 1}{T_0^2}\right\} \tag{26}$$

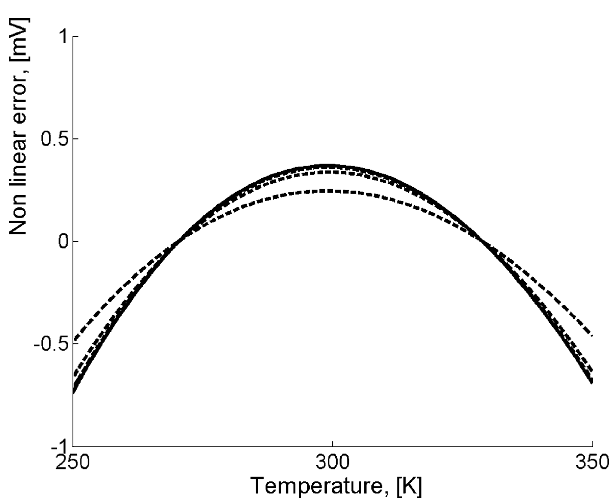

Fig. 2. The non linear error as a function of the temperature for a constant collector current (solid line) and for different PTAT/R collector currents (dashed lines; the TC of the resistor varies from 0 up to 5000 ppm/K); the Meijer parameter η_M has been considered equal to 3.

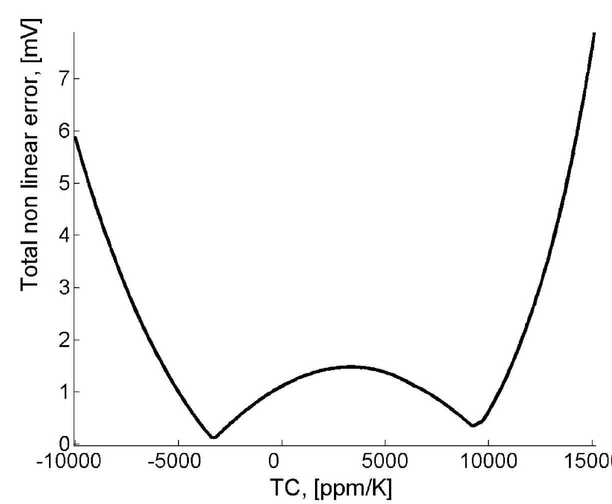

Fig. 4. The total non linear error (in the temperature range [250, 350 K]) as a function of the TC of the resistor used to generate the PTAT/R collector current; the Meijer parameter η_M has been considered equal to 3.

This relation may be used to identify the "best" solution among (18) from the point of view of better reduction of the third order non linearity errors (the solution with minimum $|\partial^3 V_{BE}(T_0)/\partial T^3|$ should be chosen).

In conclusion, the two solutions which zero the curvature are not completely equivalent, since if we take into account other non idealities (such as the spread of TCs, the non linear temperature dependence of resistances, the higher order derivatives of $V_{BE}(T)$, ...) one of those solutions is preferable. Although we have shown the theoretical method for the selection of the "best" choices (from the point of view of the spread of TCs, the non linear temperature dependence of resistances, the higher order derivatives of $V_{BE}(T)$, ...), in practical applications, it is better to use an analog simulator which is able to take into account many other non idealities which may introduce additional non linearities (such as the finite current gain of the transistor, parasitic base and emitter resistors, ...).

Finally we give a design guideline for the optimal curvature correction obtainable by using a generic collector current of the form

$$I_C = \frac{V_0(1+\alpha(T-T_0))}{R_0[1+\beta(T-T_0)]} \tag{27}$$

If there are one or more resistor types satisfying the condition

$$\beta \leq \frac{1-\sqrt{\eta_M}}{T_0} \quad \text{or} \quad \beta \geq \frac{1+\sqrt{\eta_M}}{T_0} \tag{28}$$

(a) The "best" among those resistor types must be selected (use an analog simulator).
(b) α must be tuned so that it is equal to the "best" (use an analog simulator) among the two values.

$$\alpha_{1,2} = \frac{1 \pm \sqrt{1-(\eta_M + 2\beta T_0 - \beta^2 T_0^2)}}{T_0} \tag{29}$$

If, as it is usually the case, there is no resistor type satisfying the condition (28) then

(a) PTAT/R collector current must be used.
(b) The resistor type having the "best" TC must be selected (use an analog simulator).

In the second case only a partial curvature correction may be achieved; however this is obtained "for free" (without additional components, calibration or increased design complexity) just by properly selecting the resistor type to be used in a standard bandgap circuit.

As we discussed, a practical obstacle to curvature correction is the lack of resistors with large absolute values of the TC; in the next sections we will discuss how to circumvent this technological limitation.

3. Implementation of virtual resistors with large absolute values of the TC

$\partial^2 V_{BE}(T_0)/\partial T^2$ could be zeroed if TCs of resistances may be arbitrarily tuned; however, although technological solutions for tuning TCs of integrated resistances exist, in most cases TCs of the basic resistor types are fixed (by the given process).

It is possible, in principle, to design resistive (one port) networks constituted by resistors of different types; the TC of the final resistive (one port) network may be tuned but the maximum absolute value of this TC may not be very large (see later). For this reason we introduce two different bandgap circuit topologies.

3.1. Arbitrary resistive networks

Since resistors with different TCs are generally available, it is possible to realise a resistor by using arbitrary connections of different basic resistor types. If, for instance, two resistors R_1 and R_2 are connected in series (the analysis of TCs of two series or shunt connected resistors has yet been reported in [21]), we have

$$\begin{aligned} R(T) &= R_{10}[1+\beta_1(T-T_0)] + R_{20}[1+\beta_2(T-T_0)] \\ &= R_0[1+\beta(T-T_0)] \end{aligned} \tag{30}$$

where

$$R_0 = R_{10} + R_{20} \qquad \beta = \frac{R_{10}\beta_1 + R_{20}\beta_2}{R_{10}+R_{20}} \tag{31}$$

If two resistors R_1 and R_2 are shunt connected, we have

$$R(T) = \frac{R_{10}[1+\beta_1(T-T_0)] \times R_{20}[1+\beta_2(T-T_0)]}{R_{10}[1+\beta_1(T-T_0)] + R_{20}[1+\beta_2(T-T_0)]} \tag{32}$$

and it is evident that, in the general case, $R(T)$ is not linear with temperature. It is however possible to approximate $R(T)$ with its Taylor polynomial of the first order, that is

$$\begin{aligned} &R(T) \simeq R_0[1+\beta(T-T_0)] \quad R_0 = R(T_0) = \frac{R_{10}R_{20}}{R_{10}+R_{20}} \\ &\beta = \frac{1}{R(T_0)} \times \frac{\partial R(T_0)}{\partial T} = \frac{R_{10}+R_{20}}{R_{10}R_{20}} \\ &\quad \times R_{10}R_{20}\frac{R_{10}\beta_2 + R_{20}\beta_1}{(R_{10}+R_{20})^2} \\ &\quad = \frac{R_{10}\beta_2 + R_{20}\beta_1}{R_{10}+R_{20}} \end{aligned} \tag{33}$$

In both cases (series and shunt connections) we may write

$$\beta = c\beta_1 + (1-c)\beta_2 \tag{34}$$

where $0 \leq c \leq 1$.

As a consequence, in both cases we find

$$\min\{\beta_1, \beta_2\} \leq \beta \leq \max\{\beta_1, \beta_2\} \tag{35}$$

(35) may be generalized as follows: the TC β of a resistor constituted by an arbitrary resistive networks where each resistor R_k has its own TC (β_k satisfies

$$\min\{\beta_1, \beta_2, \ldots, \beta_N\} \leq \beta \leq \max\{\beta_1, \beta_2, \ldots, \beta_N\} \tag{36}$$

It is then impossible, by using arbitrary resistive networks, to obtain an absolute value of the TC larger than the maximum absolute value of the TCs of the basic resistor types. As a result, if the conditions (12) may not be satisfied by using one of the basic resistor types, it is also impossible to satisfy (12) by using an arbitrary resistive network.

The parameter c in (34) may be tuned to any value $0 \leq c \leq 1$ depending on values of R_{10}/R_{20} (but not on absolute values of R_{10} and of R_{20}); on the other hand, since resistors R_1 and R_2 are of different types, tolerance on the ratio R_{10}/R_{20} is generally large, thus requiring calibration for accurate control of c (and therefore for accurate control of β).

3.2. The anti-series virtual resistor

We may in principle use a collector current

$$I_C = \frac{V_T \ln(r)}{R_1 - R_2} = \frac{V_T \ln(r)}{R} \tag{37}$$

that is a PTAT/R current (where the effective resistance R is the difference between two physical resistances). In order to stress that R is not a physical resistor, we refer to R as a virtual resistor; furthermore, since the virtual resistor is obtained as the difference between two resistors, we call this the anti-series virtual resistor.The temperature dependence of the anti-series virtual resistance R is given by

$$\begin{aligned} R(T) &= R_{10}[1 + \beta_1(T - T_0)] - R_{20}[1 + \beta_2(T - T_0)] \\ &= R_0[1 + \beta(T - T_0)] \end{aligned} \tag{38}$$

where

$$\begin{aligned} R_0 &= R_{10} - R_{20} \\ \beta &= \frac{1}{R(T_0)} \times \frac{\partial R(T_0)}{\partial T} = \frac{R_{10}\beta_1 - R_{20}\beta_2}{R_{10} - R_{20}} \end{aligned} \tag{39}$$

The TC, β, of the virtual resistor R is not forced to satisfy (35), and in principle any desired value of β (included values given by (18) which allows to zero $\partial^2 V_{BE}(T_0)/\partial T^2$) may be obtained; in fact the condition

$$\beta = \frac{R_{10}\beta_1 - R_{20}\beta_2}{R_{10} - R_{20}} = \beta_{desired} \tag{40}$$

is satisfied if the resistor ratio (at room temperature) r_0 is

$$r_0 = \frac{R_{10}}{R_{20}} = \frac{\beta_2 - \beta_{desired}}{\beta_1 - \beta_{desired}} \tag{41}$$

Since r_0 is a resistor ratio, it must be positive, that is

$$\begin{gathered} \left\{ \frac{\beta_2 - \beta_{desired}}{\beta_1 - \beta_{desired}} \geq 0 \right\} \\ \Updownarrow \\ \{[(\beta_2 \geq \beta_{desired}) \text{ and } (\beta_1 \geq \beta_{desired})] \quad \text{or} \\ [(\beta_2 \leq \beta_{desired}) \text{ and } (\beta_1 \leq \beta_{desired})]\} \end{gathered} \tag{42}$$

However, since typically the absolute value of $\beta_{desired}$ is much larger than the absolute values of integrated (physical) resistances, the conditions (42) are generally satisfied.

On the other hand a practical limit arises from (37): since the collector current must be positive, we need

$$R_1(T) > R_2(T) \forall T \in \text{TR} \tag{43}$$

which limits the possibility to obtain indefinitely large absolute values of TCs (however large absolute values of TCs may still be obtained).

We may rewrite (40) as

$$\beta = \frac{R_{10}\beta_1 - R_{20}\beta_2}{R_{10} - R_{20}} = \frac{r_0\beta_1 - \beta_2}{r_0 - 1} \tag{44}$$

Even in this case, although (41) shows that compensation of curvature only requires to fix a resistor ratio (at the temperature T_0), since resistors R_1 and R_2 are of different types, the accurate control of their ratio, r_0, requires calibration.

A possible CMOS circuit which accomplishes (37) is shown in Fig. 5; in this circuit a regular PTAT voltage is added to the base to emitter voltage of a transistor whose collector current is given by (37).

The operational amplifier establishes the relation (for simplicity we assume perfect matching of $M1$ and $M2$ and we neglect the base currents)

$$V_{EB1} + R_1 I_C = V_{EB2} + R_2 I_C \tag{45}$$

that is

$$V_T \ln\left(\frac{I_C}{I_{S1}}\right) + R_1 I_C = V_T \ln\left(\frac{I_C}{I_{S2}}\right) + R_2 I_C \tag{46}$$

or

$$V_T \ln\left(\frac{I_{S1}}{I_{S2}}\right) = V_T \ln(r) = (R_1 - R_2) I_C \tag{47}$$

which gives (37).

Clearly it is fundamental to make sure that, in the operative conditions range TR,

$$R_1(T) > R_2(T) \forall T \in \text{TR} \tag{48}$$

We notice that if R_1 and R_2 are (almost) linearly dependent on temperature, the anti-series virtual resistor R also shows an (almost) linear temperature dependence.

Finally we observe that the proposed circuit is a generalization of a very standard CMOS bandgap reference (which is obtained by just setting the resistor R_2 at zero).

3.3. The anti-shunt virtual resistor

We may in principle use a collector current

$$I_C = \frac{V_T \ln(r)}{R} = V_T \ln(r) \frac{R_1 - R_2}{R_1 R_2} \tag{49}$$

that is a PTAT/R current; in this case the virtual resistor R is given by

$$R = \frac{R_1 R_2}{R_1 - R_2} \tag{50}$$

so we refer to it as the anti-shunt virtual resistor.

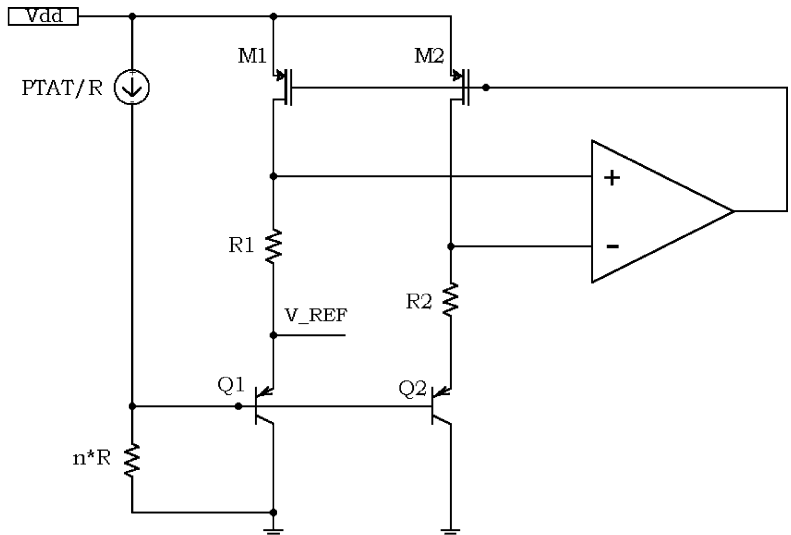

Fig. 5. Bandgap reference circuit for the implementation of an anti-series virtual resistor.

The temperature dependence of the anti-shunt virtual resistance R is given by

$$R(T) = \frac{R_{10}[1+\beta_1(T-T_0)]R_{20}[1+\beta_2(T-T_0)]}{R_{10}[1+\beta_1(T-T_0)] - R_{20}[1+\beta_2(T-T_0)]} \cong R_0[1+\beta(T-T_0)] \quad (51)$$

where

$$R_0 = \frac{R_{10}R_{20}}{R_{10}-R_{20}}$$
$$\beta = \frac{1}{R(T_0)} \times \frac{\partial R(T_0)}{\partial T} = \frac{R_{10}\beta_2 - R_{20}\beta_1}{R_{10}-R_{20}} \quad (52)$$

In this case, it is possible to impose

$$\beta = \frac{R_{10}\beta_2 - R_{20}\beta_1}{R_{10}-R_{20}} = \beta_{\mathrm{desired}} \quad (53)$$

by fixing the resistor ratio (at room temperature) r_0

$$r_0 = \frac{R_{10}}{R_{20}} = \frac{\beta_1 - \beta_{\mathrm{desired}}}{\beta_2 - \beta_{\mathrm{desired}}} \quad (54)$$

The circuit in Fig. 6 may be used to obtain a PTAT/R collector current with an anti-shunt virtual resistor. We mention that two different PTAT/R current sources are necessary, since the resistor R_1 and R_2 are in general different (then it is not possible to use a single PTAT/R current source and a current mirror). Clearly it is fundamental to make sure that, in the operative conditions range TR, we have

$$R_1(T) > R_2(T) \forall T \in \mathrm{TR} \quad (55)$$

We notice that, in contrast with the case of the anti-series virtual resistor, even if R_1 and R_2 are linearly dependent on temperature, the anti-shunt virtual resistor R may show a non linear temperature dependence, which clearly may affect the non linearity of $V_{\mathrm{BE}}(T)$.

Finally we observe that the proposed circuit is a generalization of a very standard bandgap reference (which is obtained by just replacing the resistor R_1 with an open circuit).

3.4. Effects of the spread of the TCs of the resistors

The spread of the TCs of the resistors R_1 and R_2 will introduce an error in the TC of the virtual resistor; as an example in the case of the anti-series virtual resistor we have

$$\beta = \frac{R_{10}\beta_1 - R_{20}\beta_2}{R_{10}-R_{20}} = \frac{r_0\beta_1 - \beta_2}{r_0 - 1} = c_1\beta_1 - c_2\beta_2$$
$$\mathrm{d}\beta = \frac{\partial\beta}{\partial\beta_1}\,\mathrm{d}\beta_1 + \frac{\partial\beta}{\partial\beta_2}\,\mathrm{d}\beta_2 = c_1\,\mathrm{d}\beta_1 - c_2\,\mathrm{d}\beta_2 \quad (56)$$

From Fig. 4 we see that, even if we take into account such tolerance, it is still possible to strongly reduce the curvature.

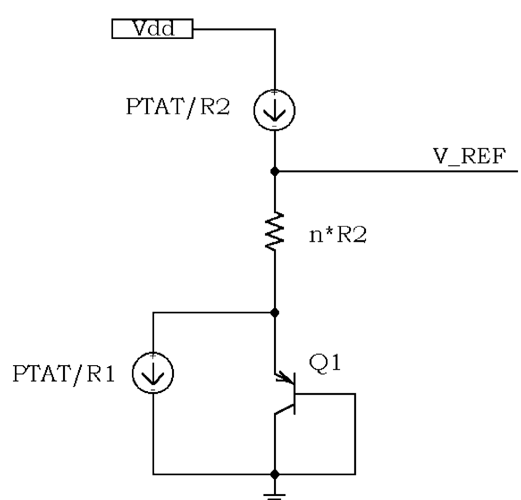

Fig. 6. Bandgap reference circuit for the implementation of an anti-shunt virtual resistor.

4. Single temperature auto-calibration procedure

It has been shown that tuning of TC is achieved by tuning of r_0, so the following single temperature calibration may be applied to all cases:

1. at $T = T_0$ we inject the currents I_X and $r_0 I_X$ through the resistors R_1 and R_2.
2. we tune the integrated resistance R_1 (for instance using a digitally tunable resistance or laser trimming) until the voltage drops across the two resistances are equal (those two voltage drops equate when $R_{10} I_X = r_0 I_X R_{20} \Leftrightarrow R_{10}/R_{20} = r_0$)

The errors in equating the voltage drops across the two resistances (that is in fixing the ratio r_0) and spread of the TCs of the basic resistor types will result in errors of the final TC, and thus in non exact zeroing of $\partial^2 V_{BE}(T_0)/\partial T^2$; if spread of TCs of available resistors and accuracy in fixing r_0 are known, the resulting total non linear error may be evaluated by designers by plots similar to that shown in Fig. 4.

5. Conclusions

Since the second order non linearity of $V_{BE}(T)$ is often the fundamental limit to thermal stability of bandgap references, curvature correction of $V_{BE}(T)$ is a main issue in the design of high performance integrated sensors systems. Although, among many other techniques, it has yet been reported that PTAT/R collector current and temperature dependence of resistances may reduce the curvature of $V_{BE}(T)$, the potentialities and the limits of these techniques were not yet systematically analysed, leading to non-optimal designs.

In this paper we have theoretically demonstrated that the use of PTAT/R collector current is generally convenient and that proper choice of the resistor type to be used in standard bandgap circuits may significantly reduce the curvature of $V_{BE}(T)$ "for free" (without calibration, additional circuitry or increased design complexity).

Furthermore, since better curvature correction is forbidden by the lack of integrated resistors with very large absolute values of the TC, we have introduced two novel low cost bandgap circuit topologies which overcome this technological limitation by mean of a virtual (anti-series or anti-shunt) resistor; finally we have discussed the effects of the spread of the TCs of integrated resistors in the proposed circuit topologies and a single temperature auto-calibration procedure.

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